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RATIONAL CHOICE

1. INTRODUCTION

THE present article [1] attempts to give an analysis of the concept of rational choice. It will be shown that rationality of choice can be characterized by criteria which are completely general and which are therefore applicable to any conceivable choice, no matter whether or not it is based on explicit standards of evaluation, no matter on the basis of what standards, if any, the choice is being made, and no matter whether or not such standards are considered as capable of objective validation. The problem of finding objective criteria for the rationality of choice will thus turn out to be independent of the question whether there are objective criteria for the rationality of standards of evaluation. Our analysis should therefore be acceptable to both objectivists and subjectivists. This issue has recently been so much in the forefront of discussions in ethics, that we tend to forget the important areas in which adherents of both schools are bound to agree.

By '*potential action* for a given person at a given time' let us understand a single act or a course of action which that person has the power to perform at that time but from which he may also refrain. 'A is now a potential action for me' means that it is now possible for me either to do or not to do A.

The *effect-complex* of a potential action A is the class of all events which would actually occur if A were to be performed.

We shall use the word '*intent*' for the presumptive effect-complex of a presumptive potential action. If I now presume that A is a potential action of mine, my intent of A includes those causal effects of A which I believe would occur if I enacted A.[2]

Someone's *potential goal* is any part of his intent which he con-

[1] I am grateful to Professor Carl G. Hempel of the Philosophy Department at Yale University for having made many suggestions from which I have benefited in the writing of this article.

[2] For the sake of simplicity, we shall assume here that all cause-effect relations are of a strictly deterministic rather than of a probabilistic character. The term 'intent' is borrowed from C. I. Lewis. "We shall include under 'intent' the *whole* expected result of the act" (*An Analysis of Knowledge and Valuation*, LaSalle, Ill., 1946, p. 367).

siders choosing. 'G is now a potential goal for me' means that I now believe that there exists some potential action for me whose effect-complex includes G, and that I will decide to bring about either G or some alternative event or state of affairs. Someone's *actual goal* is any part of his intent which he actually chooses. 'G is now my actual goal' means that G is now a potential goal of mine which I have decided to pursue.

Thus, whereas the terms 'potential action' and 'effect-complex' designate possible future *occurrences*, expressions such as 'my intent of A' and 'my (potential or actual) goal' refer to what I *presume* to be possible future events or states of affairs, hence to certain *beliefs* which I have the disposition to entertain.

2. WARRANTED INTENTS

Whether or not my decision to pursue a goal G by doing A is rational, depends on my answers to questions such as the following: Is A a potential action for me? Does A's effect-complex include G? What would be the other effects of A? What actions could I perform instead of A? What are their respective effect-complexes? The question arises whether these answers must be true in order for my choice to be rational. Is it either a necessary or a sufficient condition for the rationality of my choice that what I *believe* to be my alternative potential actions are precisely those which are *in fact* open to me, and that their intents (i.e., what I believe to be their effect-complexes) correspond to the state of affairs which would in fact occur if I performed any one of them? If so, the rationality of my present choice would depend on future happenings. If we want our rational reconstruction of 'rational choice' to conform to the way we ordinarily understand this term, we must consider my decision to be rational or irrational independently of what *will* in fact occur later and of what *would* in fact happen as a result of my making some alternative choice. Whether my choice is rational depends exclusively on factors present at that time, namely on whether my presumptions as to which alternative actions are open to me are *warranted* beliefs, and on whether their respective intents are *warranted.*

We shall consider a person's belief at a given time to be warranted if it is well confirmed by the total relevant evidence which is in fact available to him at that time, i.e., which is either in his possession or which he could obtain by suitable effort under the circumstances. If my choice is to be rational, my prediction as to what actions are open to me, and their respective intents, must con-

stitute warranted beliefs.[3] My intents would be unwarranted, and hence my choice irrational, if I failed to take into account some relevant evidence which is actually in my possession, or if I neglected to get some important additional information which I could secure by appropriate effort, or if I considered all available evidence but drew incorrect inferences.[4] It would be impossible for me, however, to hold warranted beliefs about *all* events which would actually occur as a result of my enacting *each* of the courses of action open to me. To arrive at a rational decision, it is sufficient to make warranted predictions about the *significant* effects of one's *significant* alternative potential actions, in a sense to be explained in section 5. If, on the other hand, my intents are warranted in this sense, my choice is rational regardless of whether it is successful, i.e., whether its implementation will actually bring about the state of affairs which I predicted.

3. COMPARATIVE EVALUATION OF INTENTS

Choosing involves not only predicting but also evaluating. If I am faced with the decision whether to pursue a given goal, my choice will usually be determined by my evaluation, not merely of that goal but also of each of the other events which I believe would occur if I implemented that choice. If I have to decide whether to attend tonight's concert, and if I foresee that I would in this case have to buy tickets and be unable to finish some work, my decision is likely to be influenced by my evaluation of all these anticipated possible occurrences no less than by my appraisal of my potential goal itself. My evaluation pertains, not to the goal, but to the *intent* of some presumptive potential action (e.g., buying tickets) which includes that goal.[5]

[3] C. I. Lewis calls an action *practically justified* "if and only if the intent of it is an expectation which is a *warranted empirical belief*" (*op. cit.*, p. 371). Rational choice requires that not only the action I choose but also all my alternative potential actions are, in this sense, practically justified; and even this is only one of the necessary conditions of a rational choice, as will be shown.

[4] Cf. R. Carnap, *Logical Foundations of Probability* (Chicago, 1950), p. 253: "For X to act rationally means to learn from experience and hence to take as evidence what he has observed. It means further that he should avoid considering only a biased selection from his experiences and disregarding any available information that might be relevant; therefore, we assume that he takes as basis the total evidence e available." We would specify: available to him under the circumstances. Thus, a decision to operate, taken by a physician in a hospital, may be irrational when a similar choice, made by a layman in the wilderness, is rational.

[5] Notwithstanding Everett Hall's claim that states of affairs or facts cannot be valued. "If they cannot be named, how can we ever predicate

Furthermore, I evaluate that particular intent, not in absolute terms, but in comparison with the intents of alternative presumptive potential actions. I choose, not what I value to such and such a degree, but what I prefer to what I think I could choose instead. Evaluations in the context of choice can thus always be described in comparative terms. We need not, therefore, at least with respect to evaluations of whole intents, deal with the problem of whether it is at all possible to set up any kind of value-calculus. According to our interpretation of the chooser's deliberation, he is not asking, "How much do I value attending tonight's concert?" nor, "How do I rate the whole intent of my buying tickets to that concert?" nor, "Do I prefer to go to the concert or to finish my work?" but, "Which of the two alternative intents is, as a whole, more valuable to me: buying tickets *and* enjoying the music *and* not getting my work done, or saving the money *and* foregoing the concert *and* finishing my work?" To take another example: "It is a baffling task, almost exceeding human capacity, to compare the value of an increment of national security with the value of human lives."[6] The task of the foreign policy maker, when faced with the decision whether to aim at strengthening the national security to a certain extent, is not to determine the value, or the value to him, in absolute terms, (1) of that increment of national security, and (2) of so and so many human lives which would be saved otherwise. He needs only to answer the somewhat less baffling question: "Which of the two alternative intents is on the whole more valuable to me, the intent of increasing the national security to a certain extent, which includes among its significant elements the loss of a certain number of human lives,[7] or the alternative intent which comprises leaving the nation relatively unprotected, and not endangering human lives, at least not in the immediate future?" It is generally easier to evaluate an intent as a whole than single presumptive future events separately, and to evaluate intents comparatively rather than in absolute terms. Choice involves thus, not ascribing so and so much value to some potential goal, but establishing a preference rank order among alternative intents.[8]

properties of them?" (*What is Value?* New York, 1952, p. 35). They *can* be named; e.g., the fact of my attending tonight's concert can be named by the participial phrase: 'my attending tonight's concert' (C. I. Lewis, *op. cit.*, p. 52). By means of such phrases, properties—e.g., being valuable to me—can indeed be predicated of states of affairs, actual or presumptive.

[6] A. Wolfers, "Statesmanship and Moral Choice," *World Politics*, I, no. 2 (1949), p. 191.

[7] Here the probability of the loss of (how many?) human lives would have to be taken into account; cf., however, footnote 2.

[8] It is therefore misleading to distinguish between intrinsic value-judgments pertaining to what is valuable for its own sake and extrinsic value-

4. STANDARDS OF EVALUATION

To make a rational choice, it is not sufficient that the chooser hold warranted beliefs as to which potential actions of his are significant alternatives to a given one, and that their respective intents be warranted. It is further necessary that his comparative evaluation of these alternative warranted intents be themselves warranted.

It might be held that a statement such as: 'intent I^1 is more valuable to me than intent I^2' is warranted only if it follows from a valid standard of evaluation. A standard of evaluation in the context of choice stipulates that any intent X is at least as valuable to the chooser as any other intent Y, if and only if X and Y together meet certain specified conditions. An egoistic hedonist, for example, will value any intent by virtue of, and in proportion to, its pleasurableness to him: 'For all X and Y, if X as a whole is more pleasurable to me than Y, then $X \geq Y$', where '$X \geq Y$' stands for: 'X is no less valuable to me than Y (i.e., either I prefer X to Y, or X and Y are equally valuable to me).'

The question whether normative statements such as standards of evaluation can be significantly asserted to be valid or not is at the center of the controversy between objectivists and subjectivists, the former asserting, while the latter deny, the existence of objective value properties and value standards. We need not enter here into this classic dispute, since our topic is limited to establishing criteria for the rationality of anybody's choice on the basis of his own standards of evaluation, whatever they may be. Subjectivists will claim that this is the only question which can be significantly raised in this context, while objectivists will consider our topic a limited one, whose importance they will, however, hardly deny. Indeed, a Kantian may want to know whether some particular choice of a hedonist is rational in terms of hedonism, which the Kantian would claim to be a demonstrably non-valid standard. Similarly, it may be important to know whether the Kremlin's decision to favor the attack on South Korea was rational according to the standards underlying Soviet policy. The answer will be the same, whether one adopts or rejects these standards and whether one believes, rightly or wrongly, that one can prove their validity or non-validity.

According to the analysis here propounded, a person can make rational choices on the basis of any standard of evaluation, provided only that the standard enables him to establish a linear value-rank order of all intents to which it can be applied. With respect to those

judgments about what is valuable for the sake of something else. After having made the empirical prediction that doing A would cause G to occur, I must ascertain whether or not the whole intent of A (which includes A itself as well as G) is *intrinsically* more valuable to me than some alternative intent.

intents, the relation '$\geq$' which the standard determines must be reflexive ($X \geq X$), transitive (if $X \geq Y$ and $Y \geq Z$, then $X \geq Z$), and connected ($X \geq Y$ or $Y \geq X$).[9] If I adopt a set of standards of evaluation, their conjunction must satisfy these conditions; hence, they must be compatible, i.e., they must determine the same rank order whenever they apply to the same pair of intents. Otherwise, it could happen that in making a comparative evaluation of two particular intents, I^1 and I^2, one of my standards leads me to assert that $I^1 \geq I^2$, and another that I^1 is less valuable to me than I^2. If I am an egoistic hedonist but also feel bound by certain duties toward others, and if I have to choose between one action which violates my duty but whose intent is pleasurable to me, and another action which it is my duty to perform but whose presumptive consequences I expect to give me displeasure, I cannot value-rank these two intents unambiguously by my standards unless this set includes a further rule which determines my preference rank of any two intents, one of which is more pleasurable, but at the same time less in conformity with duty, than the other.

5. WARRANTED EVALUATIONS IN TERMS OF A STANDARD

To determine whether a given intent, I^1, outranks another, I^2, according to a given standard of evaluation, is to ascertain whether I^1 stands to I^2 in that relation which the standard specifies as criterion of higher value. If, for example, I am a hedonist and have to choose whether to go to the concert or to stay home, I must determine of which of these alternative actions the intent is more pleasurable to me.[10]

To assert either of a single potential goal that it will afford me a certain pleasure, or of a whole intent that it will be more pleasurable to me than some alternative intent, is to make an empirical prediction. As such, it is either warranted or not; it is warranted under the same conditions as any empirical prediction generally and any intent in particular; and it must be warranted to make my choice rational. My choice between going to the concert and staying home will thus be rational in terms of hedonism only if my following three empirical predictions are warranted: I^1

[9] Cf. K. Arrow, *Social Choice and Individual Values* (New York, 1951), p. 13.

[10] We need not examine here whether, e.g., the relative pleasurableness of an intent as a whole can be determined with the help of some hedonic calculus, or whether 'pleasurableness' might not better be considered a Gestalt quality of an intent as a whole, and 'more pleasurable to me than' as a relation between whole intents.

is the effect-complex of my going to the concert; I^2 is the effect-complex of my staying home; I^1 will be on the whole more pleasurable to me than I^2.

Let us remember that I^1 and I^2, to be warranted, need include only the significant presumptive effects of my going to the concert and of my staying home. Relatively to the hedonic standard, a presumptive effect is significant if its presence or absence would noticeably increase or decrease the estimated pleasurableness of the whole intent. If I foresee that attending the concert will prevent me from getting my work done on time, I might consider the whole intent of my going to the concert less pleasurable than if I did not take this effect into account, perhaps even less pleasurable than staying home and finishing my work. This is therefore a significant presumptive effect; if its anticipation is a warranted belief, my choice will be rational only if I do include it in my intent-prediction. Whether or not I foresee, on the other hand, that the concert will take place in a hall built in "Baltimore and Ohio Rennaissance," will not affect my estimation of the comparative pleasurableness of the intent of my going to the concert, and hence not the rationality of my decision.

If maximization of the national security has been adopted as the only guiding value principle of a nation's foreign policy, the significant effects of any particular foreign policy decision are those, and only those, which are presumed either to strengthen or weaken the nation's security. A foreign policy decision will be rational only if the belief is warranted that its (warrantedly predicted) significant effects would, on the whole, strengthen the national security more than the intent of any alternative policy. This example also illustrates the necessity of taking into account *all* our standards. The national security standard alone would probably make it rational for the United States to become a garrison state and to transform the "free world" into its satellites. Such potential goals are, however, ruled out by the standard of democracy to which American advocates of power politics are presumably also committed. If so, an unequivocal ranking of alternative foreign policies could be obtained by incorporating the contending points of view into one total standard which stipulates generally to what extent "democracy" should be sacrificed to "the national interest" or vice versa.

Given a standard of evaluation of the form: 'whenever an intent X stands in the relation R to an intent Y, then $X \geq Y$', and a subsumption of the form: 'I^1 stands in R to I^2', the preference judgment '$I^1 \geq I^2$' can evidently be deduced. A preference judg-

ment based on an explicit standard of evaluation follows analytically from a general normative and a particular empirical statement.

My preference judgment of one intent over another is then warranted, if I have deduced it correctly from my standards of evaluation and from a warranted subsumption-prediction of these two intents under that standard. Since the relation '$\geq$' in my standards of evaluation is transitive, I can deduce the value-rank order of a series of three or more alternative intents from my preference judgments about pairs of intents.

6. WARRANTED EVALUATIONS WITHOUT REFERENCE TO STANDARDS

A given standard of evaluation provides criteria for the comparative evaluation of those intents only which are subsumable under it, i.e., which fulfill the condition-relation specified by the standard. The history of philosophy has been a continuous attempt to set up universal standards of evaluation and of value. Yet a norm as general as the Kantian provides no clue for the comparative evaluation of actions which neither correspond nor conflict with "duty" (e.g., going to the concert or staying home, if there is no work to be done). Even hedonism, which might be considered a universal norm, is hardly adopted by anyone as the sole guiding principle of all his actions. Everybody is faced with choice situations in which no single standard nor even a set of standards is available to him by which to evaluate the alternative intents. With respect to those intents, his standards of evaluation do not satisfy the condition of connectedness. Must we conclude that such choices can be neither rational nor irrational?

Whenever there exists a potential action for me, I am compelled to choose. I have the disposition to rank alternative intents according to their value for me, regardless of whether or not they are subsumable under my explicit standards of evaluation. We may therefore construe the relation '$\geq$' between two intents as being determined either by the chooser's explicit standards or by his disposition to value-rank these intents without reference to standards. The relation '$\geq$' will thus be reflexive, transitive, and connected with respect to all intents. If a pair of intents is subsumable under my standards, to evaluate them without reference to these standards might lead to a result which is incompatible with them. Thus, if the subsumption of I^1 and I^2 under my standards leads me to the conclusion '$I^1 \geq I^2$', and if I judge I^1 to be less valuable to me than I^2, my choice becomes irrational. If, on the other hand, none of my standards covers these two intents, or if there are two

or more incompatible norms under which this pair is subsumable, to prefer I^1 to I^2, to rank I^2 above I^1, or to have no preference for either intent would each make my choice rational, provided I comply with all the other requirements of rationality. If, as is likely, one cannot set up a general norm as to the comparative value of democracy and national security, one must decide in each case of conflict whether one prefers to adopt a policy which strengthens the national security or whether the principles of democracy should be upheld; in the absence of a general standard, either decision would be rational provided one's alternative intents are warranted. We often evaluate "freely" and justify our preferences afterwards by some standard which we set up *ex post facto.* Our general rules of evaluation, like those of law, evolve from particular cases.

7. CONDITIONS OF RATIONAL CHOICE

Theoretically, my choice of a goal, G, is rational only if my answer to each of the following questions constitutes a warranted belief at the time of my choice: (1) Is there a course of action open to me whose effect-complex includes G? (My choice of G is not rational unless my prediction is warranted that, all other circumstances remaining equal, G will occur if I take some appropriate action but not otherwise.) (2) Which are my potential goals which are significant alternatives to G? (3) Which are my alternative potential actions whose effect-complex includes G, and which are my alternative potential actions which would bring about each of the alternative potential goals? (4) Which are the other significant effects of each of these potential actions besides G and its alternatives, respectively? (5) Which of my standards of evaluation, if any, apply to my intents of each of these presumptive potential actions? (6) What is the preference rank for me of each of these alternative intents? (If the preference rank order is determined by some standard of evaluation, the subsumption of each pair of intents under this standard—e.g., 'I^1 will be on the whole more pleasurable (and hence more valuable) for me than I^2'—must constitute a warranted prediction.)

It is not sufficient, however, that the relevant information available to me at the time of my choice supports all these predictions. It is further necessary (1) that I hold these warranted beliefs at the time of my choice, or that I have at least the disposition to entertain them, and (2) that I do so on the basis of the available information. I may hold unwarranted beliefs and yet make the same decision at which I would have arrived on the basis of warranted predictions. I may hold beliefs which are confirmed by

the available evidence without realizing the availability of that evidence or its relevance to these beliefs. My choice would not be rational in such cases.

We are now in a position to give a definition of 'rational choice'. Someone's choice of a goal is rational if and only if the information available to him at the time of his choice and his standards of evaluation support the following belief which he has the disposition to hold at that time and on the basis of that information: 'There exists at least one course of action open to me whose effect-complex includes the actual goal and which, together with all its other significant effects, is no less valuable to me than any of the significant actions I could do instead, together with their respective significant effects.' In short: my choice of a goal is rational provided my belief is warranted that some warranted intent which includes this goal is at least as valuable to me as any alternative warranted intent.

If my intents and evaluations are warranted, and if there is an intent which I prefer to any of its alternative intents, my choice of any potential goal which is included in that, to me most valuable, intent would be rational. If there are several alternative intents which are equally valuable to me, but more so than the others, it would be equally rational to choose a goal included in any of the more valuable intents.

To determine the rationality of someone's choice, we must thus know (1) his actual goal; (2) the information available to him at the time he chooses that goal; (3) his belief dispositions at that time; (4) his standards of evaluation. Provided these elements are given to us, we are able to determine, on the basis of the criteria which have been established, whether or not any particular choice is rational.

FELIX E. OPPENHEIM

UNIVERSITY OF DELAWARE

BOOK REVIEW

Methods of Logic. WILLARD VAN ORMAN QUINE. New York: Henry Holt & Co. [1950]. xx, 264 pp. $2.90.

There was a time, say in the early thirties, when it was possible to go over the elementary parts of formal logic without committing one's self on many subtle and controversial points. The confusions which had been circulated by *Principia Mathematica* were currently avoided, at least on the continent, and the prevailing

view of logic as a kind of syntax endowed the subject with an essential simplicity. What, after all, could be more straight forward than a discussion of symbols without meaning? The progress of metalogic, particularly in its semantic aspects, has upset this happy situation so that an introductory book which sets out to prepare the ground for advanced work must deal from the very beginning with some rather fine points. *Methods of Logic* is just such a book and it performs the great service of making available to beginners a first class analysis of such notions of interpretation and validity together with a preview of the part they play in current developments.

The first two of the four roughly equal parts into which the book is divided deal with the logic of the connectives and with a comparatively simple segment of quantification theory. I shall not give them much attention except to indicate what they are intended to accomplish and how Quine has gone about the job. They are not exceptionally easy chapters but I see no reason to suppose that the same result could be achieved by a shorter route. Part III systematizes the so-called natural deduction method which originated in an attempt to bring step-by-step proof into line with the way in which people normally reason. Here I am not entirely satisfied with the machinery which has been worked out and in any case I believe it would have been advantageous to detach the explanation of deductive theory from the very difficult considerations which lead up to the proof of soundness. The possibility of making this separation is important from the point of view of the general reader who is feeling his way into the subject, as the text is much more easily handled when you know that it can be done. It would be hard to make these criticisms intelligible without telling something about the actual workings of the theory and the function which a proof of soundness performs, and this accounts for the relatively large amount of space devoted to Part III. I believe this attention could be justified, criticism or no criticism, if it helps to introduce the reader to some novel and important material. For Part IV, which sketches the expansion of logic into the region where it merges with set theory, I really have nothing but admiration. I have been rash enough to suggest that the issue of ontology has been overplayed, but whether or not this is true the text is more rather than less readable for its inclusion and in a very few pages one is able to get valuable insights into topics which are normally out of the reach of beginners.

Parts I and II are both organized around a very simple framework. The construction of skeletal sentences called *schemata* is explained and a *valid* schema is defined (refinements omitted here)

as one which remains true for all *interpretations*. Closely related notions such as *consistency, equivalence,* and *implication* are then introduced and a method is supplied for giving a yes or no answer to the question of whether they apply in any given case. Such methods, because they do so terminate, are customarily known as *decision procedures* and in this half of the book they entirely displace step-by-step proof. In Part I the schemata correspond to otherwise unanalysed sentences together with their compounds; the decision procedure consists in the so-called *truth-value analysis* explained in § 5. Subsidiary devices appear in this connection, the most elaborate being the construction of *normal schemata* which are a variant on the usual conjunctive normal form; basically these are simply ways of speeding up truth-value analysis which is capable of doing the job without supplementation. Part II introduces *uniform quantificational schemata* which include those generalized expressions which can be built up with exactly one individual variable,[1] the letter selected being "*x*". The notions of interpretation and validity have to be refined somewhat to fit this symbolism and the path to a decision procedure is more devious (since it is necessarily routed through a normal form in this case), but the ground which is covered is basically the same.

By my private scale of measurement, this material is two or perhaps three shades more difficult than anything one would ordinarily expect to find in a beginning text. Consequently the questions which arise about such an approach and which, as a matter of fact, are easily answered are likely to be based on the existence of alternatives. Truth-value analysis depends on putting "T's" and "F's" for the constituent letters of a given schema and observing (with the help of ingenious shortcuts) how it resolves under all possible combinations. A consistent schema, for example, will resolve to "T" in at least one case and the value of Quine's normal form (which I referred to as a supplementary device) lies in the fact that its consistency can be predicted for all cases. Nevertheless the old fashioned truth table will accomplish the same result and, since it is merely a tabulation of what must in any case be explained in plain English, it has a minimum of theoretical com-

[1] For technical reasons "*x*" is not referred to as a variable but merely as a letter (p. 225). Uniform schemata are not the same as monadic schemata, a point which turns out to have theoretical importance. For example "$(y)((x)Fx \vee Fy)$" is monadic in that the individual variables are not doubled up as in "*Gxy*"; it is not uniform because "*y*" occurs as well as "*x*" (p. 136). A way to construct uniform schemata is clearly described on p. 91 and it should be understood that schemata which cannot be so constructed do not count as uniform. (Explicit reference to formation rules would have made this clearer.)

plexity. Superficially, an even stronger case can be made for displacing the intricate decision procedure of Part II, for there is a well known proof that any monadic formula with k predicate letters which is valid in a domain of 2^k individuals is valid in any domain. This demonstration (which is reproduced by Hilbert and Ackermann) is fairly concise and was suggested to me on good authority as a preferable alternative. Its relevance to decisions about validity, etc., hinges on the fact that any monadic formula can be tested in a finite domain by translating universal quantifications into conjunctions and existential quantifications into alternations and then handling the result as a truth function. The proof itself does involve the use of the so-called prenex form with all quantifiers out in front and such expressions are unconstructible within uniform quantification,[2] but the text could be adjusted to this technical difficulty if the revision was urgent. Actually the alternative provides an easy way to assure ourselves of the theoretical possibility of a decision, but nothing like a test which can be applied with assurance of an easy result. The truth table suffers from the same disadvantage; we know that a result can be produced in the end but in cases of any complexity the Quine truth-value analysis is an enormous simplification. Thus a good deal can be said for the alternative approaches if the goal is simply to acquaint the reader with this part of logic in its general outlines. If any mastery of the symbolism is required, the lines on which Quine has worked out his book are clearly better. Since many of the nice theoretical issues which come up in Part III could not be very easily digested without a solid familiarity with logical formulae, I think the choice which he made was correct.

This completes what I have to say about the basic framework of Parts I and II, but there are a few additional points which should at least be mentioned. Despite the fact that this is an essentially modern approach the author does not overlook classical logic, in fact the three pages devoted to syllogistic reasoning in § 14 appear to be the shortest exposition on record which is both adequate and illuminating. Quantification symbols (combined with just one variable) are deliberately introduced at the point where they can be recognized as a way out of some of the classical limitations, and uniform quantification could be described with some justice as the theory of the supersyllogism. The appropriateness of the title hinges on the fact that the results achieved are about what would be covered by classical theory if the latter were expanded to deal both with the compounding of terms and the com-

[2] See preceding footnote.

pounding of statements. Finally I want to point out that the relating of symbols to ordinary language is by no means neglected for, throughout the book, every section which brings in new symbolic devices offers shrewd comments on their application (and misapplication as well).

Part III introduces the theory of step-by-step proof at exactly the point where checks of the kind just described are no longer available, namely the expansion of quantification theory to cover many-placed predicates (also referred to as relations). The natural deduction technique which is employed here is usually credited [3] to Gentzen and Jaskowski, who published (independently) in 1934. According to Jaskowski, whose article I saw for the first time this summer, the origins go back a little farther. "In 1926," he says, "Prof. J. Lukasiewicz called attention to the fact that mathematicians in their proofs do not appeal to the theses of the theory of deduction but make use of other methods of reasoning. The chief means employed in their method is that of an arbitrary supposition. The problem raised by Mr. Lukasiewicz was to put these methods under the form of structural rules." [4] Aside from this deliberately planned connection with ordinary ways of thinking (for the observation holds outside of mathematics as well as inside), this theory has a small and somewhat unsuspected link with the past. We are accustomed to think of modern logic as beginning with the method of axiomatization, the classical approach being confined to canons of inference describing how one statement can be obtained from another. Natural deduction systems supply no axioms and to this extent they represent a return to an earlier point of view. It is nevertheless possible to turn out theorems, in fact every "proof without premises" [5] does just this; and the demonstration that the system is sound assures us that the result is in every case valid. This calls attention to a curious historical contrast. The classical logicians found it congenial to think of their subject as making assertions "about the real," ignoring the fact that they were using a method which would exclude this. Until very recently it has been fashionable among modern logicians to talk as if the subject were "about nothing whatever," ignoring the *prima facie* evidence that its more advanced parts could not

[3] Cf. footnote to p. 25 of the 2nd edition of Hilbert and Ackermann's *Grundzüge der theoretischen Logik*. The English edition does not reproduce this. Quine's system is closer to Gentzen's than to Jaskowski's.

[4] "On the Rule of Suppositions in Formal Logic," *Studia Logica*, No. 1, p. 5. The article was published in Warsaw in 1934 but is written in English. Here "theses" could be read "theorems."

[5] *Methods of Logic*, p. 157.

be construed in this way. You will observe that the position which Quine takes at the end of this book is based on taking the upper levels of modern logic at their face value. As one would suspect, the natural deduction systems do not reach out this far, in fact Quine's rules simply yield a body of theorems which coincide with the so-called restricted predicate calculus. With one trivial exception (hinging on the fact that the rules are not expected to be workable in an empty universe) no existence theorems can be produced in this area.[6]

The possibility of using what Jaskowski called "arbitrary suppositions" has sometimes been explained by saying that the deduction theorem is being introduced as a basic rather than a derived rule. Actually, this is reproduced in a restricted form to the effect that if a conclusion C is deducible from an assumption H together with given premises, then $H \supset C$ is deducible from these premises alone. You will find that the Quine version, his so-called conditionalization rule (p. 157), is accommodated to the technicalities of his proof structure but the sense is the same. In addition, the natural deduction systems allow for dropping quantification, following out an argument in terms of free variables, and then reintroducing the quantifiers. From the point of view of "naturalness" and simplification, this last device is fully as important as the other. As already indicated, two projects are under way in Part III, the description of a deductive method and the demonstration that the method is sound in the sense that conclusions are implied by premises or, lacking premises, that they are valid. The latter occupies less than five pages; but the first half of the exposition is needed as a build-up for it, and this included the difficult topic of substitution. I will start with the theory of deduction and return to the other topics afterwards.

It begins (p. 157) with the *rule of premises* which permits the introduction of any schema [7] whatever, provided that the line so

[6] The question, what statements, if any, should be called analytic, is not mentioned in *Methods of Logic* and is thus out of bounds for a reviewer. My preference would be to restrict it to the narrower class, which involves no assertions of existence. This is the group which can most plausibly be characterized as "true by convention." Outside it we are free (as far as I am concerned) to admit the parallelism between mathematical and physical theory which Quine has used elsewhere to demolish the notion of analytic altogether. With the narrower usage, mathematics would be analytic only in a conditional sense. This last view is an old one but it was previously held without the proposed restriction on analyticity.

[7] The rules are stated in terms of schemata but I will also use the word "formula" since these schemata in fact duplicate the so-called "formulae" of the restricted predicate calculus.

introduced shall initiate a distinct column of stars on the left. All the remaining rules are stated in terms of subjoining a line to one or more which precede it; and *subjunction* is defined to mean not only the addition of a given line, but also the continuation of each previously established column of stars together with the addition of numerals on the right referring to the lines to which the new one has been subjoined. The star requirements are, in effect, a way of keeping track of relevant premises, but the matter is less obvious than might be imagined since the link between a line and its premises is not always established by implication relations. Because a decision procedure is available over the whole of the propositional calculus any step which falls in this area is taken, so to speak, in stride and referred to truth-value analysis. The *rule of truth-functional inference* (p. 157) accordingly permits the subjunction of any line which is truth-functionally implied by one or more of its predecessors. The *conditionalization rule* has already been discussed in such a way as to indicate the kind of move which is allowed. The innermost column of stars is dropped when a line is introduced in this way and the move is technically described (p. 156) as "*subjoining* a line (n) to a *line and star*, *(m)" (in distinction to the regular subjunction which continues the existing columns and deletes none).

The four remaining rules cover quantification and are stated with reference to a pair of implication relations. The first of these, called *universal instantiation*, connects "$(x)Fx$" and "Fy" (together with all variants such as "$(z)Fz$" and "Fw"); the second, called *existential generalization*, connects "Fy" and "$(\exists x)Fx$" (together with variants). The route by which these relations are introduced in the text involves the whole theory of validity and substitution; but the end result is a purely notational description of when they hold (p. 148), so that the quantification rules are really independent of this background. The first two are quite simple in that they merely permit the subjunction of schemata implied by universal instantiation and existential generalization respectively; they are accordingly abbreviated "UI" and "EG." The second two are symmetrical with the first, the result being that the direction of implication is reversed, and lines so introduced are implied by those which follow rather than those which precede them in the proof sequence. Thus the *rule of universal generalization*, abbreviated "UG", would allow "$(x)Fx$" to be subjoined to "Fy" and, more generally, any line to be subjoined to a preceding line when it implies this preceding line by universal instantiation (p. 160). Similarly, the *rule of existential instantia-*

tion, abbreviated "EI", would permit "Fy" to be subjoined to "$(\exists x)Fx$" and, more generally, the subjunction of any line which implies one of its predecessors by existential generalization (pp. 160–161).

Unlike rules UI and EG, these last call for rather careful restrictions, so that their application is only permitted in certain contexts. One of these concerns the selection of letters. The free variable "y" in the examples just cited is called the *instantial variable;* variants, e.g., the pair "Fz" and "$(\exists w)Fw$", would present a different instantial variable, in this case "z". In the application of rule UG it is required that the instantial variable be alphabetically later than any free variable previously occurring in the proof, and in the application of EI it must be later than any free variable of the subjoined schema. The other major restriction introduces a check on certain lines which are to be excluded. Whenever a line is introduced either by rule UI or EG the instantial variable must be *flagged* by writing it on the margin to the right; the rule is that no variable is to be flagged twice in the course of a deduction. (In addition, flagging is used to distinguish what are called *finished* from *unfinished* deductions (p. 164); the latter involve no incorrect step but the last line is nevertheless rejected as a consequence of those which precede it.)

Since the so-called rule of premises would ordinarily be incorporated in the definition of a proof and thus not counted at all, the score in regard to the number of rules in Quine's system should be exactly six: truth-functional inference and conditionalization plus the four which apply to quantification. These last, which are always the crucial test in systems of this kind, unquestionably have an elegance and economy which should stand comparison with any alternative likely to be produced. Prof. Turquette has claimed in a review [8] that rule EG could easily be derived from the remainder but he did not show how this could be done, and I would be willing to bet my review copy against his that no such derivation can be made without supplementing the rest of the system. My guess would therefore be that this deductive machine represents a near minimum both in respect to the number of parts and the number of moves needed to turn out a given result. The only difficulties arise when we try to understand why the apparatus works, and this immediately brings in the point of view of the teacher or popularizer of logic as well as the competent theorist. From the point of view of elementary teaching I would be inclined to hold out for a scheme in which no formula other than premises

[8] *Journal of Symbolic Logic,* Vol. 15 (September 1950), p. 203.

(or assumptions which temporarily function as premises) are admitted in a proof unless they follow from preceding lines. Rules UG and EI fail in this respect and are non-intuitive for just this reason. In addition, I would favor a set of rules in which novel technical devices were less prominent. The complete avoidance of this sort of thing is out of the question simply because the symbolism of quantification possesses a flexibility which has to be chained down at various unsuspected points. This is the price paid for effective logical tools and it seems reasonable in view of the fact that the alternative of sticking to classical logic offers nothing of much interest. There are, however, degrees of success in pushing the technical demons into the background and occasionally *Methods of Logic* seems content to allow them a free hand. The most conspicuous points have to do with flagging and the selection of letters. A case, of sorts, can be made for the necessity of flagging, although Quine does not choose to develop it in the text. You can point out that when the EI rule is used to go from, say, "$(\exists w)Fw$" to "Fz", the "z", which is the instantial variable, cannot be taken as standing for just anything but must bear a relation to the individual asserted to exist with the property F. Such being the case, you would not want to generalize on an expression conaining "z", hence you would not want it to be an instantial variable for rule UG. By flagging the instantial variable in applications of EI you indicate its special character, and when applications of UG are flagged as well you forestall the result just described. (A similar case can be made in regard to the unfinished deductions which are affected by the flagging rule.) The reader can judge for himself how far this goes towards making the device understandable and I do not want to press it very hard. On the other hand, the rule concerning the selection of letters is entirely non-intuitive and appears to work only by the intervention of providence. Quine himself does not offer precisely this explanation but he certainly makes no claim for it on the grounds of theory.

Now it so happens that this last mentioned restriction together with various other technical points are built into the Quine system in such a way as to create a fairly wide gap between a deduction described in his terms and one which is described in ordinary language. You can best check this by reviewing the definitions of premise,[9] subjunction, and the important special case of sub-

[9] There is an informal description of a premise (p. 39) as a statement which is considered to be true. Since deductions are all described in terms of schemata (which are neither true nor false) we have to fall back on the technical definition of premise as any line which initiates a column of stars (p. 156).

junction of a line n to $*m$. The result is that a student who learned this system and wished to examine a step-by-step argument with one of his untrained associates would have to improvise pretty radically before he could say anything which the latter could understand. Considering the usual objectives of an introductory course in logic, a gap of this kind is not very satisfactory. The weight which anyone will put on this kind of criticism will depend on how closely he wants to integrate deductive theory with ordinary usage. If this is not a first priority and the main objective is to provide rigorous training in modern logic, the force of the objection is pretty well reduced. As a matter of personal experience I can report that the beginning students who read *Methods of Logic* had no trouble whatever with Quine's deductive machinery, but this group was looking for just this kind of rigorous training.

There is one other feature of this system which I should like to touch on, since it involves a point of general interest. Quine's practice of incorporating truth-value analysis in his scheme of deduction (by way of the rule of truth functional inference) shortens up proofs but bypasses a good many simple rules of inference. The much discussed paradox of the material conditional hinges on the fact that "$p \supset q$" and "$\text{—}p \vee q$" are assigned the same truth conditions, and students who begin with this are sometimes led to doubt the correctness of truth-value analysis itself. Within a natural deduction system "$p \supset q$" and "$\text{—}p \vee q$" can be shown to be mutually deducible by rules which are quite simple and commonly accepted,[10] with the result that any other assignment of truth values immediately presents a counter paradox. Either we refuse the same truth values to mutually deducible statements or we must abandon one or another of a group of rules which are intuitively simple and obvious. This has not been generally recognized because the systematizations of natural deduction have not been widely circulated.

So far, I have said very little about the other topics of Part III which lead up to the proof of soundness, but I think it quite important to recognize that these can be detached from the deductive system itself; otherwise the latter will seem more complicated than it actually is. The text is not organized in such a way as to make this evident, so that the casual reader (if such there be)

[10] Besides a conditionalization rule (in some form) you need the ordinary *reductio ad absurdum* technique and de Morgan's law in a form which will yield "$\text{—}p.\text{—}q$" from "$\text{—}(p \vee q)$". In addition you need mutual implication between a statement and its double negative, implication between a conjunction and one member, and implication between any two statements and their conjunction.

might easily overlook it. The difficulties which are encountered in working up a proof of soundness center around substitution on the schematic letters "*F*", "*G*", etc., which carry the role assigned to predicate variables in most systems. (I will not enlarge on the difference between schematic letters and variables since it is irrelevant to this particular point.) Now it is a recognized advantage of natural deduction that this kind of substitution can be avoided, and there is actually no rule of substitution in Quine's system either for these or for individual variables. (The omission of a substitution rule for individual variables does not seem to me to be an advantage, as this move has so many common applications, e.g., in elementary algebra.) Consequently anyone who wanted to pass over the substitution question in order to ease the difficulties of the text for pedagogical reasons could do so without endangering anything except the proof of soundness. The resulting gap could be plugged by pointing out that the system can be proved coextensive with the usual axiomatized versions of the restricted predicate calculus. Quine notes this himself, in a footnote to p. 189, in order to show that his system is complete, and such coextensiveness will similarly show it to be sound if the standard systems are sound. This, to be sure, is a use of the "method of authority," but no one, least of all a beginning student, would be likely to doubt the assertion. There is, then, an out in respect to the more serious complications of Part III; whether you would want to take advantage of it and omit the proof of soundness is another matter.

There seem to be two possible reasons for including such a proof, one of these being the fairly obvious consideration of being sure that no conclusions will turn up which are not implied by their premises. It is easier to go wrong in this respect than might be supposed and one of the earlier Quine natural deduction schemes (published in mimeographed form) actually did so. The other reason for including the proof is that it has intrinsic interest and contributes to our understanding of the whole subject. The author makes no official choice between these two, in fact the issue is never raised, but the general tenor of the writing would indicate that he has the first reason in mind. My own conviction is that the second is the only justification for putting such really intricate material into an introductory book. The topic of substitution is notoriously confusing, as was well illustrated by the error which ran through two editions of Hilbert and Ackermann, but for this very reason it is pleasing to have an analysis which handles it in a fresh and illuminating way. The notion of predicate which is in-

troduced in this connection also plays into the explanation of validity, and this last is also a topic on which the easily accessible literature is not too satisfactory. Thus my overall conclusion is that Part III is admirably worked out, very instructive, and not in the least elementary.

Before continuing to a discussion of Part IV, I want to mention one minor error in the text, and with it a device which I found useful in connection with the proof of soundness. In its application to rules UG and EI this proof involves a recursive procedure which depends on showing that if any (finished) deduction with n flagged variables is sound in its last line, then any (finished) deduction with $n + 1$ such variables is similarly sound (pp. 164–166). To do this it is shown that a deduction can be reconstructed by way of a new premise, numbered (0), in such a way as to include one less flagged variable than the original, hence one less application of rule EI or UG. In case $n = 0$ there is no problem but the general case is hard to prove. My device, which seems to work well from the point of view of teaching, is to run through a number of examples where $n = 1$ in order to give the student a feeling for the sort of reconstruction involved. Almost all the problems and demonstrations in the text include at most one flagged variable so that examples are easy to find. You will discover, however, that the new premise (0) often fails to fit the explanation offered on p. 165. In general, the existential quantification of (0) with respect to the variable V' cannot be shown to be valid by substitution in (11) of section 25 in those cases where V' is other than "y". What is required is not merely a reference to (11) of section 25 but to alphabetical variants on this expression as well. This is a repair job which should be manageable as an exercise for students and I will not elaborate on it here. As a good illustration of the problem, take deduction 6 on p. 172, where premise (0) will be "$Fx \supset (x)Fx$" and V' is "x".

The more striking innovations as well as theoretical problems which have occupied the center of the stage since the development of modern logic arise after the subject has been extended beyond the restricted predicate calculus where quantification is limited to individual variables. In Part IV, Quine introduces the beginner to some of this material in a very concise and occasionally dramatic way. Before getting under way on these topics there are a number of sections on singular terms, and the method of dealing with this material brings up a point which I passed over in discussing Part III. Because a rule of substitution is entirely omitted in *Methods of Logic*, an argument of the familiar "All men are mortal,

Socrates is a man, hence Socrates is mortal'' type cannot be handled by putting ''Socrates'' for ''x'' in an expression containing this letter as a free variable. The way Quine deals with it hinges on the fact that his deductive theory is referred exclusively to schemata, never to actual statements. Strictly speaking, the text provides no direct way of deducing one statement from another. What you do is to construct a schematic proof the premises and conclusion of which can be appropriately interpreted and the existence of such a construction, in effect, defines the deducibility relation in its application to statements. (This is not explicitly stated in the book.) Such a usage offers no intuitive difficulty as far as theory is concerned although it is sometimes hard in actual practice to keep the intended interpretation of the same letters (''F'', ''G'', etc.) in mind from problem to problem. In handling inferences involving singular terms the same approach is used and individual variables are interpreted by names just as the schematic letters were interpreted by predicates. The deduction can then proceed in the usual schematic form (pp. 196–197). Since the interpretation of variables by way of names will not give the intended results unless the allegedly named object exists, this leads quite naturally into an investigation of the general problem of naming and the logic of descriptive phrases and identity. These topics are not easily summarized so I will merely say that exposition is beautifully concise and carried through with a minimum of semantic jargon. Having expressed general agreement and approval, a reviewer is left with the melancholy task of recording reservations and doubts which, in my case, center on the connection between names and descriptions. Aside from the manner in which the problem is explained, the end result of eliminating descriptive phrases is that of Russell and the novelty consists in doing the same thing for those singular terms which are not explicitly descriptive. Russell (at one time) contended that all proper names were, in fact, used synonymously with some description involving recognized predicates. Quine admits this for the majority of names but argues that the rest could be included by reparsing the name itself as a predicate. This is to be done ''without prejudice to epistemology'' (p. 218) since we are merely exchanging the ostensively learned predicate for the ostensively learned name. If one's epistemological prejudices happen to be such as to deny that the required predicate is always available, the reparsing is unjustified and the case falls. It seems to me unplausible to suppose that whenever a name is learned ostensively a predicate is learned ostensively as well. If this point is taken

seriously the whole topic is removed from the area of logical grammar and again becomes "shrouded" or at least entangled in the theory of knowledge and meaning contrary to Quine's expectation (p. 219). In this foggy atmosphere it is hard to be sure what is and is not plausible, but I will venture the following guess about a hypothetical individual who is dealing with the situation. First, suppose that he rejects purely ostensive names (for some reason or other). In this case it would seem unlikely that he should accept purely ostensive predicates, hence all his singular terms translate into descriptions in a fashion acceptable to Quine or Russell. Suppose, on the other hand, that he accepts purely ostensive names as meaningful (for some reason or other). In this case I contend that he has no serious problem. "Cerberus," to the best of my knowledge, was never learned ostensively; but it does present a problem of sorts in that it is meaningful, purports to name something, and in fact doesn't. This is the difficulty solved by the theory of descriptions. For the individual in question an ostensive name cannot present this problem for he has accepted it as meaningful (in some sense) and, by hypothesis, it names something. It represents, to be sure, a special kind of singular term but having accepted it I would suppose that this is the result he would expect.

The issue concerning existence and when it is presupposed in discourse is the theme which unites the explanation of the logic of singular terms with the discussion of classes and the extension of the whole subject beyond the limits of the restricted calculus. That this so-called ontology question should loom so large reflects in part the importance which the types problem has played in the history of the subject and, in addition, a phase of positivist thought in which the elimination of abstractions was considered essential to the defeat of metaphysics. The types distinction which was first regarded as the product, or at least the forerunner, of great philosophical insight has now fallen to the status of a technical puzzle for which various arbitrary and inconvenient solutions are available. Because the problem arises only when we admit assertions of class existence, the present stalemate serves to throw doubt on this presupposition. Universals as explanatory concepts in the sense of Platonic metaphysics or its medieval descendants are not currently a matter of much concern. But the nucleus of the old problem remains in the form of the question "When we appear to refer to universals (or abstract objects) do we mean it or can this be explained as a mere way of speaking?" By a sort of guilt by association process, the bad repute into which the older

theory has fallen contrived to make the second alternative appear as a liberation. Now the area where we conspicuously and unavoidably appear to refer to abstract objects is mathematics; and ever since Bernays' well known Platonism article the controversy, at least among logicians, has centered here. The great virtue of the Quine thesis relating variables of quantification with assumptions of existence is that it makes a clean cut decision possible in places where it would otherwise be easy to hedge. In this case it means that if you use the normal mathematical lingo you not only appear to presuppose but actually presuppose abstract objects. This I am prepared to accept although I may be doing so for the wrong reasons. I am prepared to believe that we mean what we say because there seems to be no harm in so doing and no great force in the existing explanations of how we can talk this way without meaning it. Isolated intuitions as to what objects are philosophically admissible or inadmissible vary from period to period and carry about the same weight as intuitions of self-evidence. On the other hand the types problem, despite the enormously important role it has played in the refinement of logical theory, ends in a technical point and it would involve a curious sense of scale to call for a wholesale revision of mathematics on such a basis.[11] I am not imputing this demand to Quine but merely indicating that I think the problem less urgent than he sometimes makes it appear.

Whether or not this attitude is justified, I am quite willing to concede that the overhauling which logical theory has received in this connection has made for improved exposition and greater insight. This can be traced throughout the concluding sections of the text which touch on classes, the absorption of mathematics into logic, the theory of types, and the Gödel incompleteness theorem. The distinction between general terms and abstract singular terms discussed in § 34 ties in with the previously established convention that the "*F*" and "*G*" of narrower quantification theory are not variables, hence not interchangeable with names. In this case one segment of logic is, as Quine puts it, freed from "the presupposition that our universe includes abstract objects" (p. 207). When classes are introduced in the enlarged calculus the additional admission of properties is rejected on the ground that abstractions would thereby be multiplied unnecesarily. (Due acknowledgment is made to the venerable Ockham.) It is even pointed out that, within narrow limits, theorems which purport to be generalizations

[11] The revision in question is not, of course, a matter of restricting class theory to get rid of contradictions but of dissolving it altogether.

about classes could be reinterpreted as assertions of the validity or consistency of certain schemata in uniform quantification theory. In this area, where we have both a decision procedure and a proof of completeness, validity and consistency can themselves be defined without reference to classes (p. 115 and p. 95 footnote). Hence these entities need never be "taken seriously" if, contrary to fact, we could limit ourselves to the theorems just referred to, and the real content of class theory is to be found in theorems lying outside this circle. Thus the question of ontology provides a dramatic theme running through the account of how logic is progressively expanded. The section on cardinal number serves at once to illustrate this expansion and to supply an easily grasped background for the types problem.

The last two sections are invaluable not only because so much is conveyed in a few pages but also because they cover the kind of material on which there is no very satisfactory reading of an elementary kind. Two axioms (A and E of § 41) are first introduced as an essential supplement to the restricted theory of quantification and their consequences are explored. More specifically, it is shown that a contradiction can be derived if they are used without restriction and that one such limiting condition is the kernel of the Russell types theory. The possibility of using alternative axioms to avoid these difficulties is examined in a section which expands and simplifies the historical note on the theory of types which originally appeared in Quine's *Mathematical Logic*. Here again, I think it is easy to trace the influence of the ontology issue; for each of these alternatives involves different existence theorems, and in a sense one decides "what there is" in choosing between them. Writers who wish to treat the Russell paradox as briefly as possible are inclined to mention the Russell solution and let it go at that, in which case the beginner is quite unaware of the alternatives and has no perspective on the matter. The question of whether any one of these axiom sets gives us all we need leads quite naturally into a discussion of the Gödel incompleteness theorem which, of course, excludes a positive answer. The exposition of these topics is superlatively good, in fact the sketch of Gödel, like that of the classical syllogism, establishes a record for brevity combined with illumination. Quine's success in these pages is due not only to an effective attack on the key points but also to a happy decision as to what should be left out. He has produced something which hangs together and is fairly easy to assimilate; if he had allowed himself the usual digressions I doubt if this result could have been achieved. The semantic paradoxes,

the branched theory of types, the reducibility axiom, the possibility of using a parallel symbolism for properties and classes, all these are more or less relevant but wholly indigestible unless treated at length, and it is well that they are omitted. There is only one point where a student might feel a gap in the sequence of ideas and this is due to a mere oversight. The axioms just considered occur in § 41—§ 42 and axioms also turned up in § 35 in connection with identity theory, but previous to this the topic of axiomatization is not mentioned by name. Actually it is discussed very briefly and effectively in the small print on pp. 191–192 in terms of proofs which depend on "a specified class of initial schemata."[12] This provides the link between the "natural deduction" of Part III and subsequent developments, but it would be easy to miss unless you knew where to look for it.

J. C. Cooley

Columbia University

NEW BOOKS

Abbott, Edwin A.: *Flatland; a Romance of Many Dimensions.* With illustrations by the author, a Square (Edwin A. Abbott (1838–1926)). Sixth edition, revised, with introd. by Banesh Hoffmann. New York: Dover Publications [1952]. 103 p. Paper $1.00; Cloth $2.25.

Albertus Magnus Lyceum for Natural Science. *Science in Synthesis; a Dialectical Approach to the Integration of the Physical and Natural Sciences.* [By] William H. Kane, Benedict M. Ashley, John D. Corcoran [and] Raymond J. Nogar, of the Order of Friars Preachers. Report of the Summer Session, July, 1952, of the Albertus Magnus Lyceum for Natural Science. River Forest, Illinois: Dominican College of St. Thomas Aquinas [1953]. 289 p. (The Aquinas Library.) $3.50.

Bahm, Archie J.: *Philosophy, an Introduction.* New York: John Wiley; London: Chapman & Hall [1953]. xiv, 441 p. $4.50.

Berkeley, George: *The Works of George Berkeley, Bishop of Cloyne.* Ed. by A. A. Luce and T. E. Jessop. Vol. Five: *Siris, Three Letters to Thomas Prior, A Letter to the Rev. Dr. Hales, Farther Thoughts on Tar-water, Varia.* Ed. by T. E. Jessop. London, New York etc.: Thomas Nelson [1953]. xi, 235 p. (Bibliotheca Britannica Philosophica.) 30 s.

[12] Under "axiom" the index refers you to p. 191 but neither this word nor any of its near relatives occurs there.

CAMPBELL, NORMAN: *What Is Science?* New York: Dover Publications [1952]. 186 p. Paper $1.00; Cloth $2.25.

CAPELLE, WILHELM: *Geschichte der Philosophie. I. Die griechische Philosophie. Erster Teil: Von Thales bis Leukippos.* Zweite, erweiterte Auflage. Berlin: Walter De Gruyter, 1953. 135 p. (Sammlung Göschen, Band 857.) DM 2.40.

CARCANO, PAOLO FILIASI: *Promblematica della filosofia odierna.* Roma, Milano: Bocca [1953]. 408 p. (Pubblicazioni dell'Istituto di Studi Filosofici—Roma. Serie II—N. 2.)

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NOTES AND NEWS

C. West Churchman has been appointed Professor of Engineering Administration and Director of the Operations Research Group at Case Institute of Technology, Cleveland, Ohio.

Lehigh University's Alfred Noble Robinson Award of $1000, given annually to a staff member not over 35 years of age and below the rank of associate professor, who has been voted as giving outstanding performance in the service of the University, was presented this year to Adolf Grünbaum, assistant professor of philosophy.

Professor Leroy E. Loemker, chairman of the department of philosophy at Emory University, will be given the honorary degree of Doctor of Laws at the University of Dubuque on June 5th. During the 1951–52 academic year he was on leave from Emory for research in Europe on Leibniz and is now preparing a critical study of Leibniz for publication.

The following fellowship awards have been announced by the John Simon Guggenheim Memorial Foundation for 1953:

Dr. Sidney Hook, Professor of Philosophy, New York University. The impact of Marxist philosophy on contemporary European thought.

Dr. Charles Frankel, Associate Professor of Philosophy, Columbia University. Studies in the philosophy of history and politics with special reference to the foundations of liberalism.

Dr. Enrico de' Negri, Professor of Romance Languages, Columbia University. A study of the secularization of the structures of Christian theology in the philosophic and economic systems of Hegel and Marx.

Dr. Heinrich Meyer, Professor of German, Muhlenberg College. Studies of Spinoza and his critics.

Dr. Arthur Walter Burks, Associate Professor of Philosophy, University of Michigan. Studies of the foundations of scientific inference.

Dr. Ruth Charlotte Barcan Marcus, Evanston, Illinois. A study in modal logic and its applications.

Dr. J. Barkley Rosser, Professor of Mathematics, Cornell University. Studies in the fields of mathematical logic and number theory.

The editorial offices of *Philosophy of Science* have been changed to the Operations Research Group, Case Institute of Technology, University Circle, Cleveland 6, Ohio. It is requested that all manuscripts be sent to that address.

The editors of *Kantstudien* announce their plans for resuming publication with Volume 45. The first issue, to appear February 12, 1954, on the occasion of the 150th anniversary of Kant's death, will be a *Gedenkband* containing scholarly contributions in English, French, and German. Future issues of the *Kantstudien* will contain the usual scholarly articles and exhaustive bibliographies and abstracts of all publications on Kant, Leibniz, and Hegel.

The monograph series (*Ergänzungshefte*) has already been resumed with Number 66.

Manuscripts for publication should be sent to the editor, Professor Gottfried Martin, Philosophisches Seminar, Universität Köln, or to Dr. I. Heidemann, Köln-Braunsfeld, Hültstrasse 32. The annual subscription is 28 DM. Orders should be placed with American importers or sent directly to the Kölner Universitätsverlag, Köln-Weidenpesch, Neusser Strasse 624. Members of the Kantgesellschaft are offered a reduced subscription price, and should place their orders with the Geschäftsstelle der Kantgesellschaft, Köln-Braunsfeld, Hültstrasse 32.